Appendix (10)
Principles and Standards of NCTM

This appendix represents the NCTM principles, the content standards and process standards, which were issued by NCTM (2000), for mathematics school.
The content of curricula documents were analyzed in light of these.

First: The principles

1.	**The Equity principle**	Excellence in mathematics education requires equity, high expectations and strong support for all students.
2.	**The Curriculum principle**	A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.
3.	**The Teaching principle**	Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.
4.	**The Learning principle**	Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge
5.	**The Assessment principle**	Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.
6.	**The Technology principle**	Technology is essential in teaching and learning mathematics, it influences the mathematics that is taught and enhances students' learning.

Second :   The Process Standards

Problem Solving	- Build new mathematical knowledge through problem solving; - Solve problems that arise in mathematics and in other contexts; - Apply and adapt a variety of appropriate strategies to solve problems; - Monitor and reflect on the process of mathematical problem solving.
Reasoning and Proof	- Recognize reasoning and proof as fundamental aspects of mathematics; - Make and investigate mathematical conjectures; - Develop and evaluate mathematical arguments and proofs; - Select and use various types of reasoning and methods of proof.
Communication	- Organize and consolidate their mathematical thinking though communication; - Communicate their mathematical thinking coherently and clearly to peers, teachers, and others; - Analyze and evaluate the mathematical thinking and strategies of others;

	- Use the language of mathematics to express mathematical ideas precisely.
Connections	- Recognize and use connections among mathematical ideas; - Understand how mathematical ideas interconnect and build on one another to produce a coherent whole; - Recognize and apply mathematics in contexts outside of mathematics.
Representation	- Create and use representations to organize, record, and communicate mathematical ideas; - Select, apply, and translate among mathematical representations to solve problems; - Use representations to model and interpret physical, social, and mathematical phenomena.

<u>**Third: The Content Standards**</u>

Number and Operations Standard

Instructional programs	Grades Pre-K–2	Grades 3–5
Understand numbers, ways of representing numbers, relationships among numbers, and number systems	- count with understanding and recognize "how many" in sets of objects; - use multiple models to develop initial understandings of place value and the base-ten number system; - develop understanding of the relative position and magnitude of whole numbers and of ordinal and cardinal numbers and their connections; - develop a sense of whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers; - connect number words and numerals to the quantities they represent, using various physical models and representations; - understand and represent commonly used fractions, such as 1/4, 1/3, and 1/2	▪ understand the place-value structure of the base-ten number system and be able to represent and compare whole numbers and decimals; ▪ recognize equivalent representations for the same number and generate them by decomposing and composing numbers; ▪ develop understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers; ▪ use models, benchmarks, and equivalent forms to judge the size of fractions; ▪ recognize and generate equivalent forms of commonly used fractions, decimals, and percents; ▪ explore numbers less than 0 by extending the number line and through familiar applications; ▪ describe classes of numbers according to characteristics such as the nature of their factors
Understand meanings of operations and how they relate to one another	- understand various meanings of addition and subtraction of whole numbers and the relationship between the two operations; - understand the effects of adding and subtracting whole numbers; - understand situations that entail multiplication and division, such as equal groupings of objects and sharing equally.	▪ understand various meanings of multiplication and division; ▪ understand the effects of multiplying and dividing whole numbers; ▪ identify and use relationships between operations, such as division as the inverse of multiplication, to solve problems; ▪ understand and use properties of operations, such as the distributive of multiplication over addition.
Compute fluently and make reasonable estimates	- develop and use strategies for whole-number computations, with a focus on addition and subtraction; - develop fluency with basic number combinations for addition and subtraction; - use a variety of methods and tools to compute, including objects, mental computation, estimation, paper and pencil, and calculators	▪ develop fluency with basic number combinations for multiplication and division and use these combinations to mentally compute related problems, such as 30 50; ▪ develop fluency in adding, subtracting, multiplying, and dividing whole numbers; ▪ develop and use strategies to estimate the results of whole-number computations and to judge the reasonableness of such results;

		develop and use strategies to estimate computations involving fractions and decimals in situations relevant to students' experience;
		use visual models, benchmarks, and equivalent forms to add and subtract commonly used fractions and decimals;
		select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method or tools.

Instructional programs	Grades 6-8	Grades 9-12
Understand numbers, ways of representing numbers, relationships among numbers, and number systems	- work flexibly with fractions, decimals, and percents to solve problems; - compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line; - develop meaning for percents greater than 100 and less than 1; - understand and use ratios and proportions to represent quantitative relationships; - develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation; - use factors, multiples, prime factorization, and relatively prime numbers to solve problems; - develop meaning for integers and represent and compare quantities with them.	develop a deeper understanding of very large and very small numbers and of various representations of them; compare and contrast the properties of numbers and number systems, including the rational and real numbers, and understand complex numbers as solutions to quadratic equations that do not have real solutions; understand vectors and matrices as systems that have some of the properties of the real-number system; use number-theory arguments to justify relationships involving whole numbers.
Understand meanings of operations and how they relate to one another	- understand the meaning and effects of arithmetic operations with fractions, decimals, and integers; - use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals; - understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve	judge the effects of such operations as multiplication, division, and computing powers and roots on the magnitudes of quantities; develop an understanding of properties of, and representations for, the addition and multiplication of vectors and matrices; develop an understanding of permutations and combinations as counting techniques

	problems.	
Compute fluently and make reasonable estimates	- select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods; - develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use; - develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results; - develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.	▪ develop fluency in operations with real numbers, vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases. ▪ judge the reasonableness of numerical computations and their results

Algebra Standard

Instructional programs	Grades Pre-K–2	Grades 3-5
Understand patterns, relations, and functions	- sort, classify, and order objects by size, number, and other properties; - recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another; - analyze how both repeating and growing patterns are generated.	▪ describe, extend, and make generalizations about geometric and numeric patterns; ▪ represent and analyze patterns and functions, using words, tables, and graphs.
Represent and analyze mathematical situations and structures using algebraic symbols	- illustrate general principles and properties of operations, such as commutative, using specific numbers; - use concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notations.	▪ identify such properties as commutative, associativity, and distributive and use them to compute with whole numbers; ▪ represent the idea of a variable as an unknown quantity using a letter or a symbol; ▪ express mathematical relationships using equations.
Use mathematical models to represent and understand quantitative	- model situations that involve the addition and subtraction of whole numbers, using objects, pictures, and symbols.	▪ model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.

relationships		
Analyze change in various contexts	- describe qualitative change, such as a student's growing taller; - describe quantitative change, such as a student's growing two inches in one year.	▪ investigate how a change in one variable relates to a change in a second variable; ▪ identify and describe situations with constant or varying rates of change and compare them

Instructional programs	Grades 6-8	Grades 9-12
Understand patterns, relations, and functions	- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules; - relate and compare different forms of representation for a relationship; - identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations	▪ generalize patterns using explicitly defined and recursively defined functions; ▪ understand relations and functions and select, convert flexibly among, and use various representations for them; ▪ analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior; ▪ understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more -complicated symbolic expressions; ▪ understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions; ▪ interpret representations of functions of two variables
Represent and analyze mathematical situations and structures using algebraic symbols	- develop an initial conceptual understanding of different uses of variables; - explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope; - use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships; - recognize and generate equivalent forms for simple algebraic expressions and solve linear equations	▪ understand the meaning of equivalent forms of expressions, equations, inequalities, and relations; ▪ write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases; ▪ use symbolic algebra to represent and explain mathematical relationships; ▪ use a variety of symbolic representations, including recursive and parametric equations, for functions and relations; ▪ judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology

	model and solve contextualized problems using various representations, such as graphs, tables, and equations	identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships; use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts; draw reasonable conclusions about a situation being modeled
Use mathematical models to represent and understand quantitative relationships		
Analyze change in various contexts	use graphs to analyze the nature of changes in quantities in linear relationships	approximate and interpret rates of change from graphical and numerical data

Geometry Standard

Instructional programs	Grades Pre-K–2	Grades 3-5
Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships	- recognize, name, build, draw, compare, and sort two- and three-dimensional shapes; - describe attributes and parts of two- and three-dimensional shapes; - investigate and predict the results of putting together and taking apart two- and three-dimensional shapes	▪ identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes; ▪ classify two- and three-dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids; ▪ investigate, describe, and reason about the results of subdividing, combining, and transforming shapes; ▪ explore congruence and similarity; ▪ make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions
Specify locations and describe spatial relationships using coordinate geometry and other representational systems	- describe, name, and interpret relative positions in space and apply ideas about relative position; - describe, name, and interpret direction and distance in navigating space and apply ideas about direction and distance; - find and name locations with simple relationships such as "near to" and in coordinate systems such as maps.	▪ describe location and movement using common language and geometric vocabulary; ▪ make and use coordinate systems to specify locations and to describe paths; ▪ find the distance between points along horizontal and vertical lines of a coordinate system
Apply transformations and use symmetry to analyze mathematical situations	- recognize and apply slides, flips, and turns; - recognize and create shapes that have symmetry.	▪ predict and describe the results of sliding, flipping, and turning two-dimensional shapes; ▪ describe a motion or a series of motions that will show that two shapes are congruent;

		▪ identify and describe line and rotational symmetry in two- and three-dimensional shapes and designs
Use visualization, spatial reasoning, and geometric modeling to solve problems	- create mental images of geometric shapes using spatial memory and spatial visualization; - recognize and represent shapes from different perspectives; - relate ideas in geometry to ideas in number and measurement; - recognize geometric shapes and structures in the environment and specify their location	▪ build and draw geometric objects; ▪ create and describe mental images of objects, patterns, and paths; ▪ identify and build a three-dimensional object from two-dimensional representations of that object; ▪ identify and draw a two-dimensional representation of a three-dimensional object; ▪ use geometric models to solve problems in other areas of mathematics, such as number and measurement; ▪ recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life

Instructional programs	Grades 6-8	Grades 9-12
Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships	- precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties; - understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects; - create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship	▪ analyze properties and determine attributes of two- and three-dimensional objects; ▪ explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them; ▪ establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others; ▪ use trigonometric relationships to determine lengths and angle measures
Specify locations and describe spatial relationships using coordinate geometry and other representational systems	- use coordinate geometry to represent and examine the properties of geometric shapes; - use coordinate geometry to examine special geometric shapes, such as regular polygons or those with pairs of parallel or perpendicular sides	▪ use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations; ▪ investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates
Apply transformations and	- describe sizes, positions, and orientations of shapes under	▪ understand and represent translations, reflections, rotations, and dilations

use symmetry to analyze mathematical situations	- informal transformations such as flips, turns, slides, and scaling; - examine the congruence, similarity, and line or rotational symmetry of objects using transformations	of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices; ▪ use various representations to help understand the effects of simple transformations and their compositions
Use visualization, spatial reasoning, and geometric modeling to solve problems	- draw geometric objects with specified properties, such as side lengths or angle measures; - use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume; - use visual tools such as networks to represent and solve problems; - use geometric models to represent and explain numerical and algebraic relationships; - recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.	▪ draw and construct representations of two- and three-dimensional geometric objects using a variety of tools; ▪ visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections; ▪ use vertex-edge graphs to model and solve problems; ▪ use geometric models to gain insights into, and answer questions in, other areas of mathematics; ▪ use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture

Measurement Standard

Instructional programs	Pre-K–2	Grades 3-5
Understand measurable attributes of objects and the units, systems, and processes of measurement	- recognize the attributes of length, volume, weight, area, and time; - compare and order objects according to these attributes; • understand how to measure using nonstandard and standard units; - select an appropriate unit and tool for the attribute being measured.	▪ understand such attributes as length, area, weight, volume, and size of angle and select the appropriate type of unit for measuring each attribute; ▪ understand the need for measuring with standard units and become familiar with standard units in the customary and metric systems; ▪ carry out simple unit conversions, such as from centimeters to meters, within a system of measurement; ▪ understand that measurements are approximations and how differences in units affect precision; ▪ explore what happens to measurements of a two-dimensional shape such as its perimeter and area when the shape is changed in some way.
Apply appropriate techniques, tools, and	- • measure with multiple copies of units of the same size, such as paper clips laid end to end;	▪ develop strategies for estimating the perimeters, areas, and volumes of irregular shapes;

formulas to determine measurements	- use repetition of a single unit to measure something larger than the unit, for instance, measuring the length of a room with a single meter stick; - use tools to measure; - develop common referents for measures to make comparisons and estimates	▪ select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles; ▪ select and use benchmarks to estimate measurements; ▪ develop, understand, and use formulas to find the area of rectangles and related triangles and parallelograms; ▪ develop strategies to determine the surface areas and volumes of rectangular solids

Instructional programs	Grades 6-8	Grades 9-12
Understand measurable attributes of objects and the units, systems, and processes of measurement	- understand both metric and customary systems of measurement; - understand relationships among units and convert from one unit to another within the same system; - understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume.	▪ make decisions about units and scales that are appropriate for problem situations involving measurement
Apply appropriate techniques, tools, and formulas to determine measurements	- use common benchmarks to select appropriate methods for estimating measurements; - select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision; - develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles and develop strategies to find the area of more-complex shapes; - develop strategies to determine the surface area and volume of selected prisms, pyramids, and cylinders; - solve problems involving scale factors, using ratio and proportion; - solve simple problems involving rates and derived	▪ analyze precision, accuracy, and approximate error in measurement situations; ▪ understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders; ▪ apply informal concepts of successive approximation, upper and lower bounds, and limit in measurement situations; ▪ use unit analysis to check measurement computations

measurements for such attributes as velocity and density	

Data Analysis and Probability Standard

Instructional programs	Grades Pre-K–2	Grades 3-5
Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them	- pose questions and gather data about themselves and their surroundings; - sort and classify objects according to their attributes and organize data about the objects; - represent data using concrete objects, pictures, and graphs	▪ design investigations to address a question and consider how data-collection methods affect the nature of the data set; ▪ collect data using observations, surveys, and experiments; ▪ represent data using tables and graphs such as line plots, bar graphs, and line graphs; ▪ recognize the differences in representing categorical and numerical data
Select and use appropriate statistical methods to analyze data	- describe parts of the data and the set of data as a whole to determine what the data show	▪ describe the shape and important features of a set of data and compare related data sets, with an emphasis on how the data are distributed; ▪ use measures of center, focusing on the median, and understand what each does and does not indicate about the data set; ▪ compare different representations of the same data and evaluate how well each representation shows important aspects of the data.
Develop and evaluate inferences and predictions that are based on data	- discuss events related to students' experiences as likely or unlikely	▪ propose and justify conclusions and predictions that are based on data and design studies to further investigate the conclusions or predictions
Understand and apply	-----------------------------------	▪ describe events as likely or unlikely and discuss the degree of likelihood

basic concepts of probability		using such words as certain, equally likely, and impossible; ■ predict the probability of outcomes of simple exp eriments and test the predictions; ■ understand that the measure of the likelihood of an event can be represented by a number from 0 to 1

Instructional programs	Grades 6–8	Grades 9-12
Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them	- formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population; - select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots	■ understand the differences among various kinds of studies and which types of inferences can legitimately be drawn from each; ■ know the characteristics of well-designed studies, including the role of randomization in surveys and experiments; ■ understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable; ■ understand histograms, parallel box plots, and scatterplots and use them to display data; ■ compute basic statistics and understand the distinction between a statistic and a parameter
Select and use appropriate statistical methods to analyze data	- find, use, and interpret measures of center and spread, including mean and interquartile range; - discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatter plots	■ for univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics; ■ for bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools; ■ display and discuss bivariate data where at least one variable is categorical; ■ recognize how linear transformations of univariate data affect shape, center, and spread; ■ identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled
Develop and evaluate	- use observations about differences between two or more samples	■ use simulations to explore the variability of sample statistics from a known

inferences and predictions that are based on data	to make conjectures about the populations from which the samples were taken; - make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit; - use conjectures to formulate new questions and plan new studies to answer them	population and to construct sampling distributions; • understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference; • evaluate published reports that are based on data by examining the design of the study, the appropriateness of the data analysis, and the validity of conclusions; • understand how basic statistical techniques are used to monitor process characteristics in the workplace
Understand and apply basic concepts of probability	- understand and use appropriate terminology to describe complementary and mutually exclusive events; - use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations; - compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models	• understand the concepts of sample space and probability distribution and construct sample spaces and distributions in simple cases; • use simulations to construct empirical probability distributions; • compute and interpret the expected value of random variables in simple cases; • understand the concepts of conditional probability and independent events; • understand how to compute the probability of a compound event